Absolute instabilities and the role of confinement in stratified shear flows

**Motivation**

Finite-amplitude manifestations of stratified shear flow (s.s.f) instabilities and their spatio-temporal coherent structures are believed to play an important role in turbulent geophysical flows. A confined s.s.f has been produced experimentally in [1]:

This flow succumbs to a left-going Holmboe instability. After saturation, an inherently nonlinear, robust and long-lived coherent structure is observed:

\[ \rho(y=0, t=0) \]

\[ -16 \quad -14 \quad -12 \quad -10 \quad -8 \quad -6 \quad -4 \quad -2 \quad 0 \]

\[ -1 \quad 0 \quad 1 \]

**Problematic**

In practice, the duct is of finite and reduced length. We may ask in consequence if the observed structure is the product of travelling instabilities reflected at the duct boundaries - in which case it is specific to this experiment - or is it intrinsic to confined s.s.f in general?

Answering this problematic calls for a spatio-temporal analysis

**Governing equations**

Duct flow is modeled by means of incompressible Navier-Stokes equations under Boussinesq assumption:

\[ \nabla \cdot \mathbf{u} = 0 \]

\[ \partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = - \nabla p + R(\cos \theta \mathbf{e}_x + \sin \theta \mathbf{e}_z) + Re^{-1} \Delta \mathbf{u} \]

\[ 0 = \partial_t \rho + \mathbf{u} \cdot \nabla \rho = (Re \cdot Sc)^{-1} \Delta \rho \]

Assuming no spanwise direction (2D flow) and small perturbations \( f = f(z) \exp(i k x + \sigma t) + \epsilon \) around:

\[ \hat{U}(z), \hat{W}(z) = (-\sin(\pi z), 0) \]

for the velocity and

\[ \hat{R}(z) = -\tanh(2 R(z - z_0)) \]

for the density, leads to a generalized eigenvalue problem:

\[ \sigma \mathbf{A} \mathbf{x} = \mathbf{B} \mathbf{x} \]

The flow is unstable if \( \exists k : \Re(\sigma(k)) > 0 \)

**Absolute/Convective discrimination**

Modes \( k_0 \in \mathbb{C} \) that dominate the long-term impulse response in the laboratory (fixed) frame have a null group velocity [2], i.e they satisfy:

\[ \exists k_0 : \Re(\sigma(k_0)) > 0 \] : the flow could be absolutely unstable and invade its whole domain.

\[ \forall k_0 : \Re(\sigma(k_0)) < 0 \] : the instability is either stable or convectively unstable. In this last case, the unstable packet disappears in the lab. frame

**Spatio - temporal analysis**

Such \( k_0 \in \mathbb{C} \) correspond to saddle points in the \( \Re(\sigma(k)) \) isocurves in the complex plane. For the left-going Holmboe wave we draw:

Integration path: goes through a saddle point if and only if it is a valid one

Neutral line (null growing rate)

As we lower the density interface with respect to the velocity centreline, the saddle point \( k_0 \) remains valid and converges toward the most unstable temporal mode: the instability becomes absolute and the impulse response tends to grow on place symmetrically.

**Sidewalls effect**

We further consider the spanwise direction and associated sidewalls in a duct of aspect ratio \( A \) :

Without spanwise direction

Sidewalls have a stabilizing effect. The 2D dispersion relation is recovered as we increase the aspect ratio of the duct.

**Conclusions**

We showed that confined stratified shear flows are absolutely unstable as soon as the offset between the density interface and the velocity base profile centreline is marked enough. Physically, this velocity then advects the gravity wave (made unstable by the vorticity one) in such a way as to reduce its group velocity. For weak stratification, confinement is responsible for such a transition to absoluteness.

Furthermore, removing the spanwise direction provides a suitable approximation for 3D instabilities as soon as sidewalls are sufficiently far, and/or the spanwise boundary layer thickness of the base flow sufficiently thin.

**References**


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