Effect of air friction on the natural frequency of an oscillating system

Motivation & Objectives
The oscillating system of a mechanical watch contains a balance wheel, on which the air friction is responsible for significant damping and frequency variations. The goal of the present work is to study the air flow around the wheel oscillating around its axis as well as its action on the wheel. The influence of parameters such as the Reynolds number and the confinement are tested.

To simplify the computations, the rim and arms are studied separately in 2D simulations.

Oscillator analysis
The dampings found by simulation are integrated into the equation of the harmonic oscillator:

\[ \ddot{\theta} + \omega_0^2 \dot{\theta} + k \theta + l \dot{\theta} \theta^{1/4} = 0 \]

This equation can be solved numerically, and studied analytically using the multiple scale analysis, allowing to calculate the quality factor.

Experiment
The quality factor of an oscillating system as a function of the amplitude is measured optically using a balisometer, which measures the reflection time of a laser on an arm of the wheel while it oscillates. Thin metal plates are inserted between the wheel and its support to change the confinement. It appears again that at small distances, the behaviour is closer to linear and the quality factor drops.