Energy Harvesting by a Flapping Flag

Context and Objectives

A plate (tipically a "flag") loaded by an axial flow is subjected to a fluid-structure instability as the incoming flow velocity is increased. When clamped-free boundary conditions are used, the system leaves the stable straight state for limit cycle oscillations. This is the so-called flutter instability. If it can be highly destructive, we can also hope for harvesting - part of - the energy provided to the plate by the fluid. Different harvesting strategy are currently under investigation: (i) using the displacement of the flag through induction or (ii) using the curvature of the flag through piezoelectric patches. A third strategy based on a rotating flagpole is numerically studied in the present work.

Model and Methods

1) Solid model : inextensible Euler Bernoulli beam with free trailing edge
\[
\frac{\partial^2 \mathbf{x}}{\partial t^2} = \frac{\partial}{\partial s} \left[ M^* \mathbf{f}_T \mathbf{e}_n - \frac{\partial^2 \theta}{\partial s^2} \mathbf{e}_n \right] + M^* \mathbf{f}_{\text{fluid}} \mathbf{e}_n
\]

at the flagpole :
\[
\frac{\partial^2 \theta_0}{\partial t^2} + N_1 \frac{\partial \theta_0}{\partial t} + \Omega_0^2 \theta_0 = N_3 \frac{\partial \theta}{\partial s} \bigg|_{s=0}
\]

2) Fluid model : a resistive term + Lighthill’s Large Amplitude Elongated Body Theory (LAEBT)
\[
f_{\text{fluid}} = -\frac{1}{2} C_d u_n |u_n| - m_a H^* \left( \frac{\partial u_n}{\partial t} - \frac{\partial}{\partial s} (u_n u_\tau) + \frac{1}{2} \frac{\partial \theta}{\partial s} \right)
\]

3) A set of non-dimensional numbers

Flag parameters : 
\[
M^* = \frac{L^2}{\mu}, \quad H^* = \frac{H}{L}, \quad u^* = \frac{U^*}{U},
\]

Flagpole parameters : 
\[
N_1 = \frac{CL}{U^*}, \quad N_0 = \sqrt{\frac{K^f L^2}{U^2}}, \quad N_3 = \frac{BL}{U^2 L^2} \mu L^2 \mu
\]

damping (harvesting)  natural frequency  coupling

Some results

1) A frequency lock-in phenomenon is observed, due to the coupling of an unstable oscillator (the flag) and a stable one (the flagpole). The harvesting efficiency is enhanced under lock-in conditions.

Harvesting efficiency : 
\[
\eta = \frac{\text{extracted power}}{\text{power of the wind passing through a section } AH}
\]

\[
\eta = \frac{N_1}{M^* U^4} N_0 \frac{\langle \dot{\theta}_0^2 \rangle}{A} \times 10^{-4}
\]

2) Some flags with non-homogeneous material or geometrical properties were tested. For example, we considered the effect of adding a localized mass (modeled as a gaussian) at some location \( s_0 \) along the flag. If the mass is located around the trailing edge, the energy harvesting efficiency can be significantly improved.

Conclusion

The occurrence and the practical interest of a lock-in phenomenon was shown. Moreover, an optimization of the harvesting efficiency (for \( M^*, U^*, H^* \) fixed) was performed, showing that our strategy leads to efficiencies slightly inferior to what is obtained with a piezoelectric based strategy. Besides, we proposed a simple way of enhancing the efficiency by adding a localized mass at the trailing edge of the flag. The technical simplicity of the flagpole based strategy compared to the other strategies should inexcitable further work on this particular topic.

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