Sampling of graph signals for graph-based view synthesis and image completion

Dion Tzamarias – Pascal Frossard
LTS4
Dion.tzamarias@epfl.ch

Introduction
We tackle the problem of sampling signals on graphs in Paley-Wiener spaces [1]. By introducing a new sampling algorithm, using the theoretical minimum number of samples, we can reconstruct perfectly the signal. In order to reduce the computational complexity of our algorithm we propose a local sampling approach, by partitioning the original graph in subgraphs and applying our sampling algorithm to each individual subgraph. We examine which graph partitioning method works best and in what cases.

Theory/Method/Hypothesis
In order to design our algorithm we make use of the following 3 observations:
• Observation 1[2]. A set of vertices S is a uniqueness set for all signals \( f \in \text{PW}_{\lambda_n}(G) \) if and only if \( q_1(S), q_2(S), \ldots, q_n(S) \) are linearly independent. Where \( \lambda_n \) is the nth smallest graph Laplacian eigenvalue and \( q_i(S) \) is the reduced eigenvector that corresponds to the ith smallest eigenvalue.
• Observation 2[2]. For any frequency \( \lambda_n \), the smallest uniqueness set \( S_{opt} \) for signals \( f \in \text{PW}_{\lambda_n}(G) \) has a size of \( ||S_{opt}|| = n \).
• Observation 3. For any minimum uniqueness set \( S \) of size \( k \) for signals in \( \text{PW}_{\lambda_k}(G) \) there always is at least one node \( S_i \notin S \) such that \( S \cup \{S_i\} \) is a uniqueness set of size \( k + 1 \) for signals in \( \text{PW}_{\lambda_{k+1}}(G) \)

Based on the following equations we propose that normalized cuts will result in a perfect signal reconstruction in the case of n-connected-component graphs using less samples than if we sampled the signal without the partitioning step. We also conclude on the hypothesis that as the graph becomes more heavily connected, local sampling via normalized cuts [3] will produce larger reconstruction errors for the same number of samples required for a perfect signal reconstruction when skipping the partitioning step (global sampling).

\[
\begin{align*}
\sum_{t=0}^{n} \lambda_i^2 x_i &= f^T L_{D_2} f + \sum_{i=1}^{n} \sum_{j=1}^{n} (f_i - f_j)^2 W_{ij} \\
L_D &= \begin{bmatrix} L_{G_1} + D_1 & W_{D2} \\ W_{D1} & L_{G_2} + D_2 \end{bmatrix} \\
\sum_{i=1}^{N} \lambda_i &= \text{trace}(L_{G_1}) + \text{trace}(D_1) + \text{trace}(L_{G_2}) + \text{trace}(D_2)
\end{align*}
\]

The reconstruction process that is used is the least square reconstruction method [4].

Results
The algorithm that we propose is the following

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Algorithm 1 Global Sampling
INPUT: n first eigenvectors \( Q \).
OUTPUT: Minimum uniqueness set \( S \).
1. \( S = \emptyset \).
2. \( S(1) = \) index of any nonzero element of first eigenvector \( q_1 \).
3. for \( i = 2 \) to \( n \) do
   4. \( q_i \) = all nonzeros of first eigenvector \( q_i \).
   5. compute \( f = \) nullspace \( Q_i(S) \).
   6. compute \( S = Q_i(f, S) \).
   7. end for
\`

Our algorithm vs algorithm proposed in [2]

Local vs Global sampling on 2-connected component graph (left) and connected component graph (right)

Conclusion/Perspectives
In this project we have developed a Global-Sampling strategy that uses the minimum number of samples in order to perfectly reconstruct a signal lying on a graph. We have shown that in some cases our Local-Sampling algorithm using normalized cuts outperforms the Global-Sampling algorithm.

Acknowledgments
We thank Professor Frossard and supervisor Pinar Akyazi for their guidance throughout the project.

References
[1] Isaac Pesenson. SAMPLING IN PALEY-WIENER SPACES ON COM- BINATORIAL GRAPHS.