Subgrid-scale Modeling of Cavitating Bubbly Flows Using an Eulerian-Lagrangian Approach

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Motivation

Cavitation is a phenomenon that can be observed in various situations ranging from hydraulic machinery to medicine. Because of its role in a wide range of applications, correctly predicting and understanding its behavior is important to improve the various processes in which it occurs. The project at hand describes the development of a coupled Eulerian-Lagrangian method to model cavitating flows at the subgrid-scale. The liquid phase is solved on an Eulerian grid while the disperse bubbly phase is computed using the Lagrangian formalism. The model is then intended to be applied to bubbles cloud simulations.

Mathematical model

On one hand, the liquid phase is described by the Euler equations, to which source terms describing the gaseous phase \( \alpha_g \) are added, in an Eulerian frame of reference:

\[
\frac{\partial p}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = \frac{\rho}{1 - \alpha_g} \left( \frac{\partial \alpha_g}{\partial t} + \mathbf{u} \cdot \nabla \alpha_g \right)
\]

\[
\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \frac{\rho}{1 - \alpha_g} \mathbf{u} \cdot \nabla \alpha_g + \frac{\alpha_g}{1 - \alpha_g} \mathbf{u} \cdot \nabla (\rho \mathbf{u})
\]

\[
\frac{\partial \rho E}{\partial t} + \nabla \cdot (\rho E \mathbf{u}) = -\nabla \cdot (\rho \mathbf{u} p) + \frac{\rho}{1 - \alpha_g} \mathbf{u} \cdot \nabla \alpha_g + \frac{\alpha_g}{1 - \alpha_g} \mathbf{u} \cdot \nabla (\rho E)
\]

On the other hand, the bubbly disperse phase is treated in a Lagrangian frame of reference:

\[
\frac{d \mathbf{x}_b}{dt} = \mathbf{u} + \mathbf{u}_s
\]

where \( \mathbf{u}_s \) is the slip velocity.

The coupling between the two reference frames is obtained from the time evolution and spatial distribution of the gaseous phase volume fraction.

Numerical implementation

Liquid phase:

- Finite volume Godunov-type scheme for the spatial discretization.
- 3rd order low storage Runge-Kutta scheme for the time integration.

Disperse bubble phase:

- Time integration with a 5th/6th order Runge-Kutta-Verner scheme.
- Pressure reconstruction to retrieve \( p_\alpha \) for the computation of the bubble oscillatory dynamic equation.

Information transfer from one frame of reference to the other:

- Lagrangian to Eulerian: Smearing of the bubbles volume onto the neighboring cells using a Gaussian distribution.
- Eulerian to Lagrangian: Gradient-based interpolation from the cells centers to the bubbles locations.

Model validation

Bubble scattered pressure:

The bubble oscillating dynamic being described by the Keller-Miksis model, it is expected to retrieve a scattered pressure in the surrounding liquid comparable to the analytical results of this same model.

Bubbles radius evolution in a small bubbles cluster:

The simulation of a bubble cluster enables to model the bubbles interactions. The simulation on the right depicts the evolution of two bubbles radii. Those are compared to experimental data and to a solution computed with a commercial Boundary Element Method (BEM) software[1].

Test cases

Sudden pressure increase:

- 250 bubbles clouds
- Bubble initial radii: \( R_0 = 0.3 \text{ mm} \)
- Initial void fraction: 7.4%
- Pressure ratios: \( p_\alpha/p_0 = 5 \) and \( p_\alpha/p_0 = 10 \)
- Maximum pressures at clouds centers: \( p_{max} = 200 \text{ bar} \) and \( p_{max} = 475 \text{ bar} \)

Single pressure pulse:

- 100 and 200 bubbles clouds
- Bubble initial radii: \( R_0 = 0.6 \mu\text{m} \)
- Initial void fraction: 0.01% and 0.02%
- Pressure pulse: \( p = 10 \cdot \sin(2\pi ft)\text{ bar} \) with \( f = 300 \text{ kHz} \)

Conclusion

The comparison of the coupled flow solver with analytical, experimental or other numerical solutions of common cavitating flow problems yields satisfactory and promising results as a large range of phenomenon is captured by the model. In particular, these include the individual bubbles volumetric changes along with the associated pressure oscillations in the surrounding liquid, and the bubble-bubble interactions. The bubbles cloud test cases simulations confirm expected trends when varying parameters such as the bubble concentration or the amplitude of the driving pressure. Moreover, the model enables modeling cavitating bubbles at the sub-grid scale, thus significantly reducing the amount of grid cells required and the associated computational cost compared to other grid-based methods.

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