Introduction

Passive circuits are generally reciprocal and have a symmetric scattering matrix, meaning transmission from a source to a load is the same when the two are exchanged [4]. This behavior is due to microscopic reversibility and is valid for any linear, time-invariant circuit [5]. Reciprocity can be broken by introducing some magnetic non-linear materials or by modulating one or more elements of the circuit in a time-periodic manner. Breaking the reciprocity using time-modulated elements rather than non-linear ones is a promising way to achieve it without suffering from the incompatibilities with integrated circuit technology [5].

Such time-modulated systems might transmit power unidirectionally and change the frequency of an input signal. For example, it could be used in non-reciprocal devices such as Dodgson’s resonator for emission and absorption. This kind of modulation is called time-Floquet, which generates harmonics separated each by the modulation frequency.

Previous Work

Previous work was conducted by [3] using Coupled Mode Theory (CMT) to analyze two resonators (resonant frequencies ω₁ and ω₂) linked by a time-modulated coupling (at frequency Ω = ω₂ − ω₁).

When resonator 1 is excited through port 1 (with decay rate γ₁) at ω₁, the time-modulated coupling transfers the energy to the second resonator at frequency ω₂ + Ω = ω₂, which resonates in this second resonator. This circuit is also non-reciprocal. When port 2 is excited, the frequency in resonator 1 is ω₂ + Ω ≠ ω₁, which does not resonate.

This circuit was analyzed in [3] using CMT. This approach supposes the coupling of the resonators together and to the ports can be treated as a perturbation. Finite Differences Time-Domain (FDTD) is a numerical simulation method that does not require this approximation and can bring more accurate results.

FDTD Method

Many numerical methods were investigated in this work, however, only the best performing one is exposed here. The three fundamental elements of the circuit (resistor, capacitor, and inductor) do not behave the same when time-modulated:

\[
V(t) = R(t)f(t) \\
\frac{dL[T(t)]}{dt} \quad \Leftrightarrow \quad L(t) = \int_0^t V(t')dt' \\
\frac{d[C(T(t))]}{dt} \quad \Leftrightarrow \quad C(t)V(t) = \int_0^t I(t')dt'
\]

These integral equations can be translated in their FDTD equivalent using linear multistep methods such as Adams-Moulton or Backward Differentiation Formula (BDF) [6].

\[
\frac{L^k_{k+1}}{Δt} \sum_{i=0}^{k} a_i L^{k-i}_{k+1} V_{k-i} + R^k_{k+1} I_{k+1} = 0 \\
\frac{C^{k+1}}{Δt} \sum_{i=0}^{k} a_i C^{k-i} V_{k-i} - \sum_{i=0}^{k} b_i I_{k-i} = 0
\]

where coefficients a_i and b_i depend on the linear multistep method chosen.

The unknown variables in these discretized equations are the voltage and currents across each element. They can be expressed using only the node voltages and the mesh currents, which therefore are the state variables of any arbitrary circuit. By including another equation for each voltage source, one has enough equations to formulate them in a matrix form and solve the state variables \( V_{k+1} \) and \( I_{k+1} \) recursively.

Results

The electrical circuit chosen to model the coupled resonators from \( \mathcal{A} \) is depicted below.

![Electrical Circuit Diagram]

It is composed of two LC resonators coupled by the time-modulated resistor and capacitor \( R(t) \) and \( C(t) \). The four coupling modulation cases investigated in \( \mathcal{A} \) are:

\[
K(t) = t_0 + \Delta k \cos(\Omega t) \quad \Leftrightarrow \quad (R(t) = R_0 [1 + m \sin(\omega_m t)] \\
C(t) = C_0 [1 + m \sin(\omega_m t)]
\]

When \( K(t) = k_0 + \Delta k \exp(j\Omega t) \) and \( K(t) = \Delta k \exp(j\Omega t) \) with \( \Delta k < k_0 \) and \( K(t) = \Delta k \exp(j\Omega t) \), the circuit was unstable because \( t = R(t)c(t) < 0 \). Indeed, the voltage of a capacitor discharged in a resistor is proportional to \( \exp(-t/\tau) \) which is stable only for \( \tau > 0 \). However, the other modulations where close to the results predicted by the CMT analysis. Power ratio between the load port and the excitation port for different configurations are reported below:

<table>
<thead>
<tr>
<th>Modulation</th>
<th>Feeding</th>
<th>Power transfer</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>Port 1 0 ( \omega_1 )</td>
<td>1.53 dB</td>
</tr>
<tr>
<td>( b )</td>
<td>Port 1 0 ( \omega_2 )</td>
<td>-7.01 dB</td>
</tr>
<tr>
<td>( c )</td>
<td>Port 1 0 ( \omega_2 )</td>
<td>-25.8 dB</td>
</tr>
<tr>
<td>( d )</td>
<td>Port 1 0 ( \omega_2 )</td>
<td>-21.0 dB</td>
</tr>
<tr>
<td>( e )</td>
<td>Port 1 0 ( \omega_2 )</td>
<td>-9.18 dB</td>
</tr>
</tbody>
</table>

Case \( a \) is almost reciprocal. It will never be totally reciprocal because the coupling capacitor frequency dependence behavior and that the two ports have different frequencies. Cases \( b \) and \( c \) are clearly non-reciprocal and have opposite coupling directions. \( b \) corresponds to \( \Omega > 0 \) and \( c \) to \( \Omega < 0 \). Also, in case \( a \), parametric amplification happens: there is more power absorbed in port 2 than given by port 1 because the capacitance modulation brings energy to the system.

Conclusion and Future Work

An efficient and polyvalent FDTD simulation method has been developed using Matlab. It has been used to simulate without using CMT the system investigated by \( \mathcal{A} \). However, the electrical circuit equivalent to this system has to be improved yet: it is not truly reciprocal and is also unstable in some cases. Nevertheless, it was sufficient to show most of the properties predicted in the previous work. Also, a physical prototype can be built to confront the theory to the measurements.

Acknowledgments

This work is the achievement of personal effort and willingness. Nevertheless, it could have been impossible without the precious help of two different persons. I therefore wish to sincerely thank Theodoros Koutserimpas and professor Romain Fleury who followed my work and were of a great help.

References